

AI and the Future of Polyhedral Research (2026 edition)

Jesús A. De Loera

Joint work with

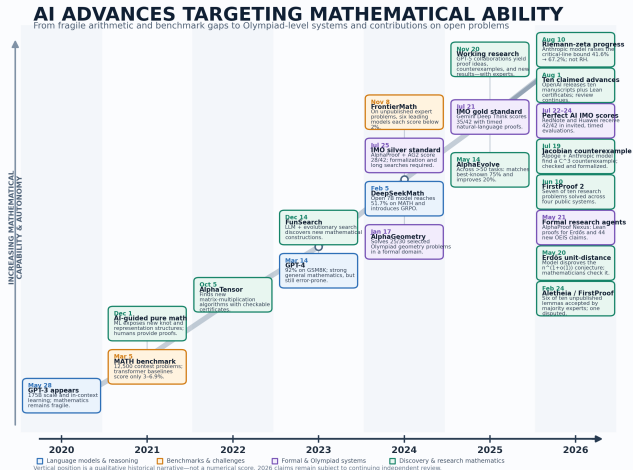
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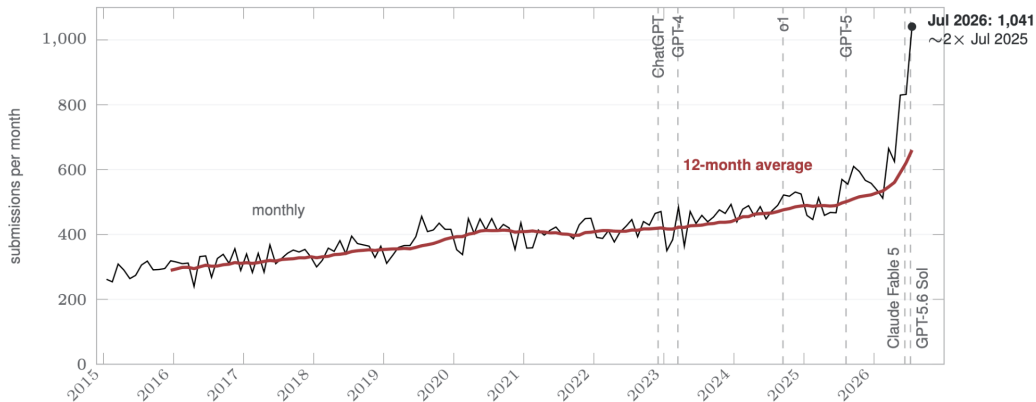
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LLMs are Getting Better at Doing Mathematics ...



arXiv math.CO submissions



Monthly submissions to arXiv math.CO by first-version date (arXiv API, retrieved Aug 2026).

AI can now credibly complete most undergraduate assignments, MIT warns

A new report urges rapid changes to college teaching, learning and assessment as the technology advances.

August 25, 2026

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Has AI solved MATH?? Will mathematicians go extinct?



yes, nonsofic groups exist: this statement is one of many new beautiful results proved by Astra, our next major model.

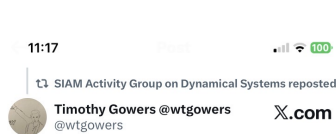
We're releasing 10 such Astra proofs, complete with lean certificates and CoT walkthroughs for each of them. The results are wide-ranging, from von Neumann algebras (disproof of Connes' Rigidity Conjecture) to better bounds for high dimensional sphere packing, for circuit complexity, for monochromatic triangles in multicolored graphs, and more.

More thoughts here:



1:34 AM · 8/1/26 · 3.5M Views

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But if AI mathematics continues to progress at anything like its current rate -- which is what I expect to happen -- then we will face a crisis very soon, and mathematics departments, who owe a duty of care to their students, should be urgently preparing for it.

12:20 PM · 5/8/26 · 249K Views

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What shall we believe?

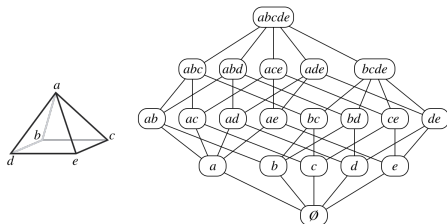
- What is the level of success of AI in Math? What are its limits?
- What is the ability in average? We hear only of successes from AI! How about failures?
- Has AI fully solved even ONE single area of math already?
- I have been working for 30 years in polyhedral geometry, I think I can judge AI's abilities with confidence...

- We are a team from Duke U., Harvard U., UC Davis, NSF AIMING grant 2434665:
“AI Tools to Knowledge Discovery and Rigorous Reasoning in Polyhedral Geometry.”
- **Goals:**
 - ① Use AI to advance research in polyhedral geometry (with and without AI). E.g., methods of machine learning to search for interesting examples. Data collection we do *automatic generation of conjectures*. LLM's for idea/proof brainstorming.
 - ② We seek *automated reasoning on polyhedral geometry*. We use Integer/Linear programming proof systems and algebraic geometry proof system in automatic proofs of statements.
 - ③ *Polyhedral mathematics as a benchmark to test AI capabilities*. Benchmark has 2 levels:
 - ① Grad student-level 3-dimensional polyhedral questions only,
 - ② A collection of open problems we have attacked with AI's help.

Polyhedra touch ALL Mathematics, Nice Theorems too!!

Example invariants

- algebraic properties (symmetries, embeddings, alg. varieties)
- combinatorial properties (f -vectors, graphs, etc)
- metric properties (volumes, projections/sections, integration, etc)



Lots of nice theorems:

- **4-Color theorem** (solved but no human proof yet!)
- **Euler-Poincaré:** $f_0 - f_1 + f_2 - \cdots + (-1)^{d-1} f_{d-1} = 1 + (-1)^{d+1}$.
- **Upper Bound Theorem:** Among d -polytopes P with $f_0(P) = n$, the cyclic polytope $C(n, d)$ has the most i -faces.
- **Ehrhart's Theorem:** The function $f(k) = \#(kP \cap \mathbb{Z}^d)$ is a quasipolynomial of degree $\dim(P)$ and leading coefficient $\text{vol}(P)$

Polyhedral Research is a Perfect Test for AI!!

Computation friendly

Algorithms: Convex hull, Triangulations, Mixed Integer Programming, etc Software: OSCAR / Polymake / Normaliz / LattE, etc

Datasets: Classification of smooth Fano polytopes, empty simplices, tropical fans

Automation Friendly

Flyspeck (Kepler)

Automated proofs of realizability (Bokowski et al).

LP, SAT, Gröbner tools popular tools

Popular to Search and Enumerate (e.g., Firsching, TOPCOM, AlphaEvolve)

Powerful Math Theory

Strong links to Algebraic Geometry, Commutative Algebra, Rep. Theory

Strong Links to functional analysis, Probability

Strong Links to Combinatorics (e.g., Fields Medal 2022 (June Huh) heavy on polyhedral theory).

What is C3PoSt?

C3PoSt = **C**ertified **3**-dimensional **P**olyhedral **S**tatements

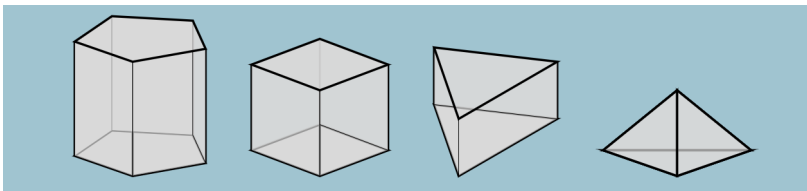
C3PoSt is a focused mathematical repository of 1st year grad student level. Solvable problems that:

- Includes **hundreds** of automatically statements about the geometry and combinatorics of **3D convex polyhedra, polyhedral complexes**. We can generate new problems without too much effort!
- For each proposition we have automatically produces either a **rigorous proof** or an **explicit counterexample**.
- Our automated proofs are found using **linear/integer programming and algebraic geometry proof systems**.
- Our constructions require explicit geometric/topological operations and symbolic computing.
- What type of polyhedral problems are in this benchmark? What Prompts do we ask the LLMs??

The p -vectors of a 3-polytope P

$$p(P) = (p_3, p_4, p_5, \dots)$$

- p_k represents the number of k -gon faces in P .
- We often truncate at a largest p_m . It is the **p -vector** of the polytope.



EXAMPLE: $(0,5,2)$, $(0,6,0)$, $(2,3,0)$, $(4,0,0)$

LLM Prompts: CAN YOU CONSTRUCT SUCH A POLYTOPE?

Conjecture

Let a sequence $(p_3, p_4, p_5) = (4, 0, 0)$ be the p -vector of some simple convex 3-polytope. Can this sequence also be realized with a simple 3-polytope with exactly 7 hexagons?

IMPOSSIBLE! By work of Jucovic and Jedrol.

Conjecture

Let a sequence $(p_3, p_4, p_5) = (4, 0, 0)$ be the p -vector of some simple convex 3-polytope. Can this sequence also be realized with a simple 3-polytope with exactly 8 hexagons ($p_6 = 8$)?

YES! but construction is not obvious and it is difficult for LLMs.

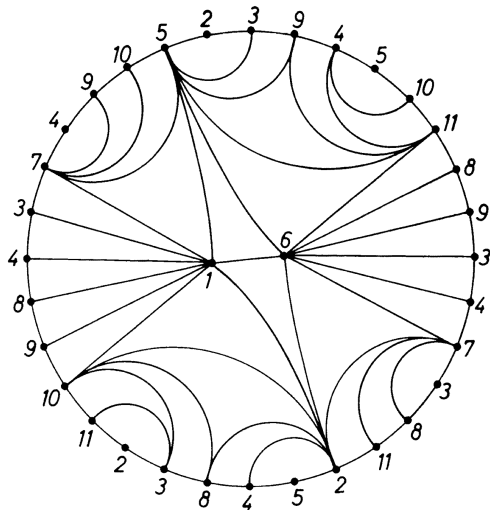
LLM Prompt: What do you see in this picture??

What do you see in this picture?

- A disk subdivided into topological triangles.
- 32 boundary vertices and 2 interior vertices (labeled 1 and 6).
- Triangle count for a triangulated disk:

$$T = b + 2i - 2 = 32 + 2 \cdot 2 - 2 = 34.$$

NOTE: Only CHATGPT 5.4 can read this information correctly from the picture alone.



Identifying equally labeled boundary sides...A surface

Quotient complex gives a surface!

Identify boundary edges carrying the same labels with opposite orientation.

Repeated boundary labels collapse to an vertex set $\{1, 2, \dots, 11\}$.

Euler characteristic

$$V = 11, \quad E = 51, \quad F = 34,$$
$$\chi = V - E + F = 11 - 51 + 34 = -6.$$

Edge pairings reverse orientation

$$(5, 2) \sim (2, 5), \quad (2, 3) \sim (3, 2), \quad (3, 9) \sim (9, 3), \dots$$

Hence the quotient surface is orientable.

Conclusion

This is a closed orientable surface,

$$\chi = 2 - 2g.$$

Therefore

$$2 - 2g = -6 \implies g = 4.$$

Piecewise-linear realization problems in $\mathbb{R}^3$

Prompt to LLM

Can one find coordinates $x_1, \dots, x_{11} \in \mathbb{R}^3$ for each of the 11 vertices such that the quotient complex is realized as a straight-edge triangular surface? If it is possible provide explicit coordinates, if not possible provide a proof of impossibility.

Geometric requirements

- 1 The realization has the correct topology.
- 2 Every face in the list is a nondegenerate Euclidean triangle.
- 3 No two triangles intersect improperly.

PL-embedding condition

For any simplices σ, τ in the complex,

$$|\sigma| \cap |\tau| = |\sigma \cap \tau|.$$

So two triangles may meet only in a common vertex or a common edge.

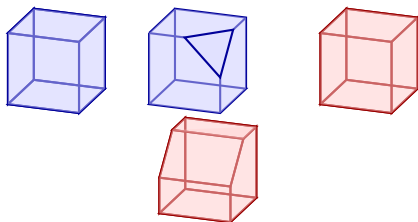
STATUS: LLMs tried to build a numerical search for coordinates and check for improper embedding. No certified coordinate set yet and does not decide whether to keep trying or try to prove is impossible.

How do we verify/certify all our statements???

We rely on known constructive algorithms and alternative proof systems! such as Linear/Integer Programming Proof systems and Algebraic Geometry Proof systems

How to search the space of polytopes?

Chopping operations that **preserve** simple
3-polytopes:



- Steinitz and Rademacher and many others developed **geometric surgery** for 3-polytopes. But it requires strategy and geometric intuition on how to apply them.
- We can attack conjectures and find examples with such operations.

allow us to **explore** the entire space.

How do we grade the LLMs' performance??

Benchmark Structure & Release Policy

First Release

- **85 public** statements released with paper.
- Additional **100 private** statements queryable by external users.
- Target size: **500** statements.

For each statement the LLM responds with:

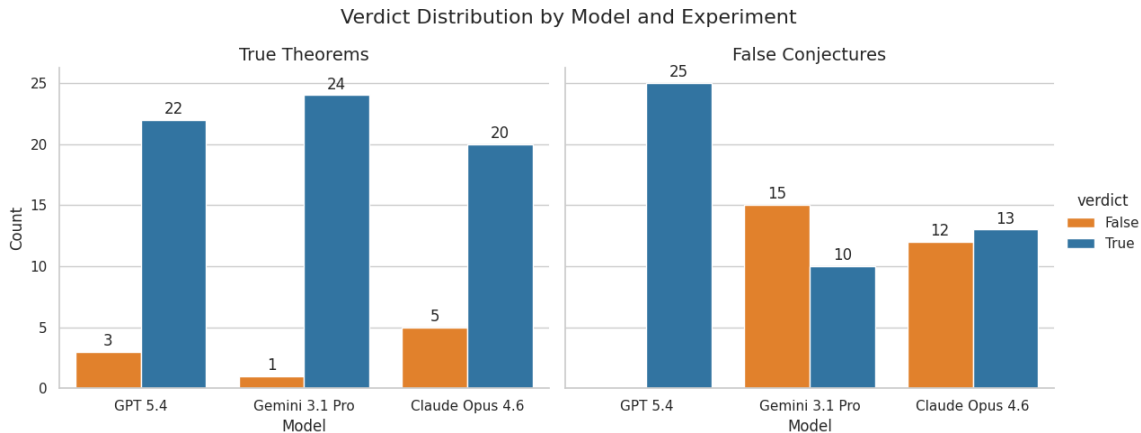
- 1 A **TRUE / FALSE** declaration.
- 2 A full **proof or explicit counterexample**.
- 3 An explicit **certainty probability** (e.g. "I am 62% certain of my answer").

Set UP Measures

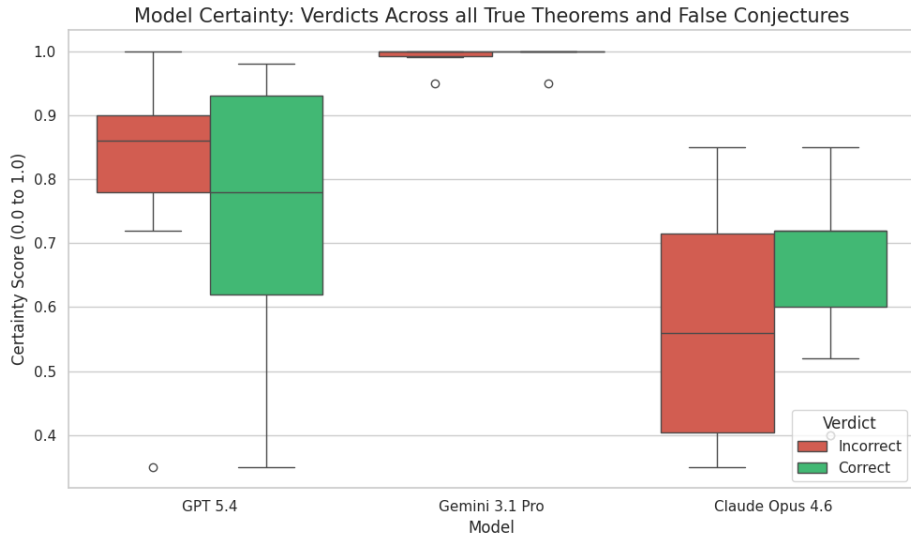
- Split into *public* / *private* / test sets.
- Reproducible logs (we save prompts, tokens used, time) required.

We let LLMs operate with access to **web search** and access to mathematical software (e.g. Python SageMath), a realistic research environment.

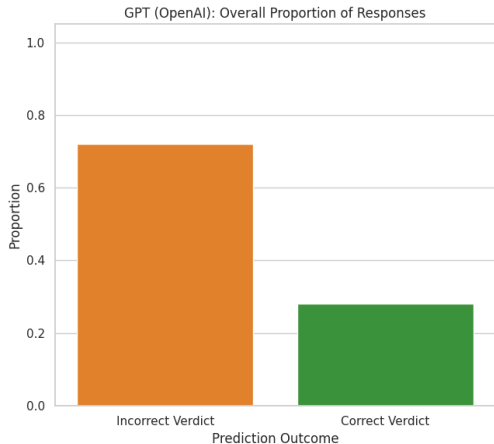
Accuracy on the answers



Coherence of thought



25 constructions of specific polytopes



CONCLUSION: The performance of lower-level LLMs is NOT spectacular!

LLMs' performance on research-level problems

Challenge 1: Chudnovsky-Nevo problem

Question: What is the maximum size of an independent set in the graph of a simplicial sphere?

$\beta(d, n)$ = maximum over all simplicial $(d - 1)$ -spheres on n vertices.

$\alpha(d, n)$ = maximum restricted to flag $(d - 1)$ -spheres on n vertices.

Theorem

- For every $d \geq 4$, $\beta(d, n) = n - \Theta(n^{1/\lfloor d/2 \rfloor})$.
- For flag spheres, for every $d \geq 4$, $\alpha(d, n) \geq n - 4\sqrt{n} + O(1)$.
and $\alpha(d, n) = n - \Theta(\sqrt{n})$ for dimensions $d = 4, 5$.
- For Bier spheres in n vertices, independence number cannot be larger than $\lfloor \frac{n}{2} \rfloor$.

Challenge 2: Problems on faces

Conjecture (Cube-simplex Kalai's Conjecture 1990)

For every positive integers k, ℓ , there exists an integer $f(\ell, k)$ so that every polytope of dimension $\geq f(\ell, k)$ has a ℓ -dimensional face which is a simplex or k -dimensional face is combinatorially isomorphic to a cube.

Notation: $f_s(\ell, k)$ denote the threshold restricted to simple polytopes.

Finiteness of $f(\ell, k)$ is known only for $\ell, k \leq 2$. Kalai proved that $f_s(2, k) \leq 2k^2$.

Theorem

- $f_s(\ell, k)$ is finite for all $\ell \geq 2$ and $k \geq 3$, with $f_s(2, k) \leq 2k^2 - 1$ and $f_s(\ell, k) \leq \frac{1}{2}k^2\ell 2^k$ for $\ell \geq 3$.
- $f(\ell, k) \geq (5\lfloor \ell/2 \rfloor + (\ell \bmod 2) - 1)(k - 1) + 1$ and $f_s(\ell, k) \geq \max\{4, 2(\ell - 1)\}(k - 1) + 1$.

Theorem Every convex polytope of dimension at least 15 has a 3-face with at most 13 facets.

Challenge 3: The p -vectors SIMPLE 3-polytope P

Simple 3-dimensional polytope: each vertex is adjacent to exactly 3 edges (also 3 facets).

Theorem (Eberhard 1888)

If $(p_3, p_4, p_5, p_7, \dots, p_m)$ is a p -vector of a simple 3-polytope satisfies

$$\sum_{k \geq 3} (6 - k)p_k = 12. \quad (1)$$

Conversely, if a sequence satisfies Eq 1, then there exists a value p_6 such that $(p_3, \dots, p_m)$ is the p -vector of some simple 3-polytope.

OPEN Questions:

- (A) For which p_6 is a sequence realizable by a polytope?
- (B) What are good upper and lower bounds of p_6 for simple 3-polytopes?
- (C) How many non-isomorphic simple 3-polytopes exist for a given 3-realizable sequence?

Work by Thurston, Grünbaum, Barnette, many others

Challenge 4: A Problem on Lattice Points

Conjecture (Integer Radon Problem)

For $d \geq 1$, define the integer Radon number $r_{\mathbb{Z}}(d)$ to be the smallest integer N such that for every lattice polytope of N vertices, its vertex set can be partitioned into two nonempty sets A, B for which the polytopes

$$\text{conv}(A) \cap \text{conv}(B) \cap \mathbb{Z}^d \neq \emptyset.$$

Find the exact value of $r_{\mathbb{Z}}(d)$. In particular, is $r_{\mathbb{Z}}(3) = 11$

The best known bounds for general d are $5 \cdot 2^{d-2} + 1 \leq r_{\mathbb{Z}}(d) \leq d(2^d - 1) + 3$. For $r_{\mathbb{Z}}(3)$ Bezdek Blockhuis proved less than 17.

Theorem: For $r_{\mathbb{Z}}(3) \leq 15$. We have also found many new examples showing $10 \leq r_{\mathbb{Z}}(3)$.

We tried multiple search methods for counterexamples (including AlphaEvolve and others).

Challenge 5: Fatness of 4-dimensional polytopes

The fatness of a 4-polytope P is $(f_1 + f_2)/(f_0 + f_3)$ (f_i = number of faces of P of dimension i .)

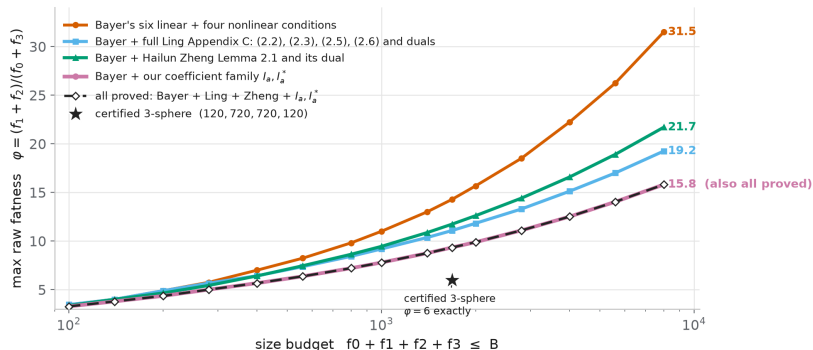
Question: Does there exist a constant c so that every convex 4-polytope has fatness $\leq c$?

What is known: False for spheres!! If it exists for polytopes, how big is c ?

Rigorous 4-polytope fatness upper bounds vs. size

Only proved necessary conditions are shown. Blue and green add full Ling and Hailun Zheng separately to the Bayer baseline.

Purple isolates Bayer + our I_a, I_a^* . Black diamonds add every proved family and coincide with purple at all sampled budgets.



Challenge 6: Kalai's 3^d conjecture

Conjecture

Every centrally symmetric d -polytope P satisfies the number of non-empty facets of P (including P) is at least 3^d .

Equality holds for Hanner polytopes, Kalai conjectured (up to linear images) they are the only minimizers.

Our progress: Google Deepmind asserted true for $d = 5$, but gives a 400 page proof!!

Do we believe it? We have to digest it! An overview seems fine, but there are too many details to check. Not sure of correctness!! **What to do with this?**

Long proofs beyond human readability!! Formalize with the help of LLM's!!

Challenge 7: Hirsch conjecture, Diameter Bounds

Conjecture (Hirsch Conjecture 1957)

For the graph of a polytope $\text{diam}(P) \leq f_{d-1} - d$.

False!! Santos solved it in 2010, but today open in dimensions 4, ..., 18. Deepmind team found 19 dimensional example in 2025 (following closely Santos 2015 paper, no LLMs, just Reinforcement learning).

Conjecture (Polynomial Hirsch)

For the graph of a polytope $\text{diam}(P)$, is bounded by a polynomial in the number of facets and the dimension.

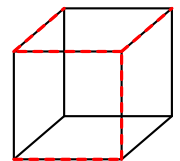
Our progress: we have been able to build an 18 dimensional counterexample (humans slightly improving Deepmind's example). No other progress! No new ideas! Not sure I would publish this, but it needs to be recorded. Zero progress on Polynomial Hirsch.

MANY more Challenges of Intuitive Geometry questions failed!!

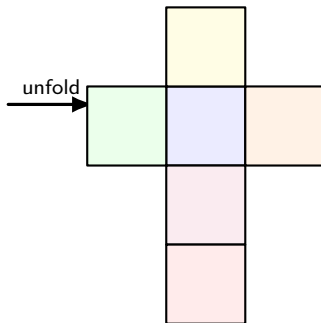
Intuitive visual problems in low dimensions are extremely hard for LLMs!!

Conjecture (Dürer's Conjecture)

Every convex polyhedron admits a non-overlapping unfolding



convex polyhedron



cross-shaped non-overlapping net

Conclusions/ Reflections /Possible actions

- **AI is very very useful! Makes things fast!** Yes, ML and LLM allow for lots of new theorems, writing useful code, allowing for math to try more avenues of research. It made cheap and accessible to experiment. LLMs know the entire literature and can work on tedious tasks more diligent than humans.
- **AI has not conquered polyhedral research, it shows limitations** Even top results appear mostly a recombination of a larger literature and better searching for examples and experimentation. E.g. Erdős unit distance is not a “from scratch” originality. Quality depends on the model (number of tokens?) Terry Tao “LLMs can jump, but cannot climb”. On average LLM’s are still not amazing. Moreover, proofs from LLMs are often long, winding roads of correct arguments.
- **We are proving the same quality of papers, but A LOT more of them** We have an abundance of facts, but a scarcity of understanding and certainty on those facts. Autoformalization helpful, but may not be enough!! If AI proves something but nobody understand it, is it proven?

Non-trivial obstacles for LLMs to “dominate” Math

- (0) AI revolution in a way started by winning games! Math is infinite, it is not a game like Chess or Go. It does not even have a single set of specified rules (think **Gödel’s undecidability!!**).
- (1) The cost to produce or verify answers can be excessive! Not only costly on tokens energy etc but also on computational complexity. See **My Diophantine Polytopes game**. See also **Realization spaces of Convex Polytopes NASTY!!**
- (2) Recombination of existing results has a limit. E.g., Alphafold has largely plateaued from AF2 to AF3 no improvement. Data is limited in Math, gains only come from new human input, new ideas, new preprints. All it is trained in our papers. **There is no evidence it will go beyond existing human math through recombination.**
- (3) LLMs lack long-term memory and planning. Stochasticity still leads to incoherences and hallucinations. Math is not just about text symbol processing either! E.g., humans can learn from pictures or video, how to replicate spatial or geometric intuition?

[Submitted on 25 Aug 2022]

Turning Mathematics Problems into Games: Reinforcement Learning and Gröbner bases together solve Integer Feasibility Problems

Yue Wu, Jesús A. De Loera

Can agents be trained to answer difficult mathematical questions by playing a game? We consider the integer feasibility problem, a challenge of deciding whether a system of linear equations and inequalities has a solution with integer values. This is a famous NP-complete problem with applications in many areas of Mathematics and Computer Science. Our paper describes a novel algebraic reinforcement learning framework that allows an agent to play a game equivalent to the integer feasibility problem. We explain how to transform the integer feasibility problem into a game over a set of arrays with fixed margin sums. The game starts with an initial state (an array), and by applying a legal move that leaves the margins unchanged, we aim to eventually reach a winning state with zeros in specific positions. To win the game the player must find a path between the initial state and a final terminal winning state if one exists. Finding such a winning state is equivalent to solving the integer feasibility problem. The key algebraic ingredient is a Gröbner basis of the toric ideal for the underlying axial transportation polyhedron. The Gröbner basis can be seen as a set of connecting moves (actions) of the game. We then propose a novel RL approach that trains an agent to predict moves in continuous space to cope with the large size of action space. The continuous move is then projected onto the set of legal moves so that the path always leads to valid states. As a proof of concept we demonstrate in experiments that our agent can play well the simplest version of our game for 2-way tables. Our work highlights the potential to train agents to solve non-trivial mathematical queries through contemporary machine learning methods used to train agents to play games.

Subjects: **Machine Learning (cs.LG)**; Optimization and Control (math.OC)

Cite as: [arXiv:2208.12191](https://arxiv.org/abs/2208.12191) [cs.LG]

(or [arXiv:2208.12191v1](https://arxiv.org/abs/2208.12191v1) [cs.LG] for this version)

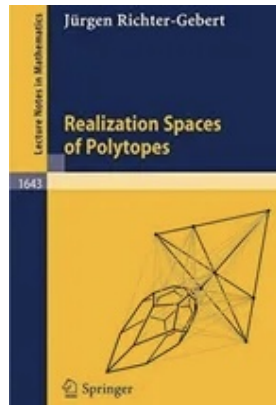
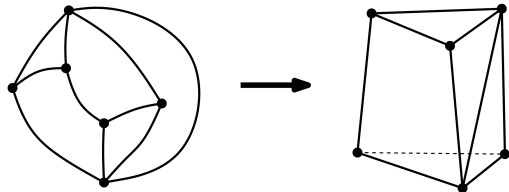
<https://doi.org/10.48550/arXiv.2208.12191> 

Submission history

From: Yue Wu [\[view email\]](#)

Realizing Planar Graphs as Convex Polytopes

Günter Rote
Freie Universität Berlin



Rethinking Being a Mathematician

- **Our workflow, reward system, and infrastructure will have to change.** Go back to the time where long monographs/books were written about a subject, not just papers. The literature is being inundated by publications, many are undigested. We need ways to control the tsunami of slop! Publishing and refereeing were in a crisis even before AI. Slop should not be deposited in the Arxiv.org, there should be an OrphanMath.org.
- **Overflow of “mathematical facts” is being produced by AI and humans.** Maybe we have abundance of proofs, but proofs alone are not enough to advance knowledge. So much to do after the fact! One needs to *Verify, Condense/Summarize, Rank/Organize, Explain/Simplify facts to obtained understanding*. How to deal with this overload? More experts? Formalization?
- **Follow the norms of international math, be humane, take responsibility of your papers!:** Use AI but follow the scientific method, present your results to make them verifiable and reproducible openly. Allow proper scrutiny of results. Find and credit the sources that made a result possible. Companies are producing slop and abandoning the rest of the job of making it understandable. Take responsibility your paper is correct!

Let's learn from Biology!

-Biology has gone through many technology revolutions in one century, more than mathematics!!(gene editing, protein folding structure, etc). It took a Ph.D to sequence one gene, now machines take weeks.

-But Biologist did not go extinct! Researchers to work on other things:

- ① validation, verification, reproducibility,
- ② higher level of knowledge, meta-analysis,
- ③ improve accuracy, confidence, robustness
- ④ find new applications (a bounty for applied math!!!),
- ⑤ modeling new phenomena for prediction,
- ⑥ interpreting results, hypothesize,
- ⑦ extend research to broader settings, develop new technologies, etc, etc, etc, etc!!

AI IS NOT THE END OF MATH, LOTS TO DO AT EVERY EPOCH!!
AND WHEN A NEW SMARTER LLM COMES WE CAN MOVE 1 UP